A Demographic Model. DNpension

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Dirección de Estudios Económicos

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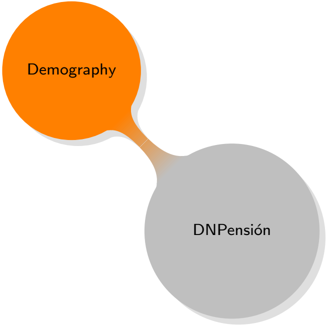
DNPension Model

DNPensión

DNPension Model



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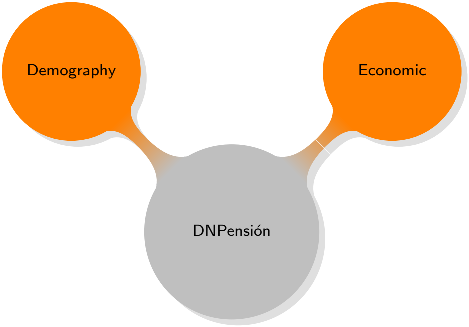

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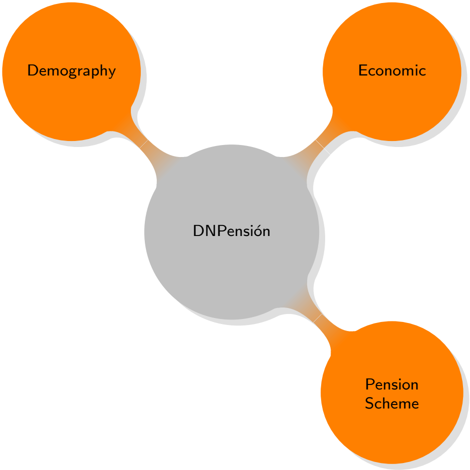

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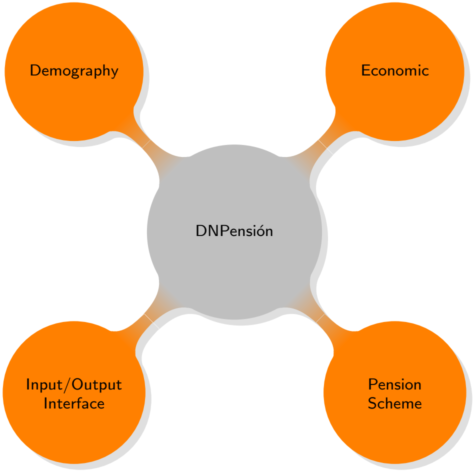


Forecasting population

Methods

- Deterministic.
- Stochastic (Uncertainty)

Variables

- Fertility.
- Mortality.
- By age, gender and education

Our proposal

Method and variable

- Stochastic.
- Using education as key variable.
- Age-patterns

Facts

Better educated adults

- Lower mortality rates.
- Their children - better changes of survival.

Women with higher levels of education

- Fewer children through choice and higher access to birth control.

Potential improvements in education have significant implications for future population growth

Education

Measure

- Flows
- Stocks

Education

Flows

- Process of education "Production of human capital"
- School enrolment rates
- Student-teacher ratios
- Drop-out ratios
- Repetition rates

Education

Stocks

- Human capital (Stock of educated adults)
- Result of past education flows.
- Quantity of formal education (highest level)
- Quality dimension (General knowledge, cognitive skills)

Lee and Carter (1992)- Journal of the American Statistical Association

The Lee-Carter model was originally designed to forecast age-specific mortality rates with the following specification

$$log\, \mu_{x,t} = \alpha_x + \beta_x \kappa_t + \xi_{x,t} \tag{1}$$

$$\kappa_t = \phi + \kappa_{t-1} + \varepsilon_t \tag{2}$$

Where x denotes age. t : time
$\mu_{x,t}$: Mortality rate. α_x : Age profile
β_x : How fast the rates decline over time in response of changes of κ_t
κ_t : time-specific effect.
$xi_{x,t}$: Error term. Assumed $\sim N(0, \sigma^2)$
To ensure identifiability of the model parameters $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$

General framework for forecasting mortality, fertility, emigration and immigration A. Wisniowski(2015)

First - Lee-Carter model with Poisson variability of death counts Bayesian framework.Czado et al.(2005)

- Assume that count data on a given population component follow a Poisson distribution

$$Y_{x,t}^{g,k} \sim Poisson(\mu_{x,t}^{g,k} R_{x,t}^{g,k}) \tag{3}$$

Where $g \in D, B, E, I$. D represents deaths (mortality), B is births (fertility), E is emigration, I is immigration. k denontes gender. $k = F$ denotes females, and $k = M$ for males. $\mu_{x,t}^{g,k}$: Counts/Risk Population. $R_{x,t}^{g,k}$: Population exposed to risk of these events. For D, E, population risk is the same. For B population risk are the women of reproductive age. For I is hard know risk population, forecast for counts rather than rates. They assume that $R_{x,t}^{I,k} = 1 \forall x, t, k$

General framework for forecasting mortality, fertility, emigration and immigration A. Wisniowski(2015)

Second. They assume that the logarithm of the rate follows a normal distribution

$$log\,\mu_{x,t}^{g,k} \sim \mathbf{N}(\alpha_x^{g,k} + \beta_x^{g,k}\kappa_t^{g,k} + \gamma_{t-x}^{g,k}, \tau^{g,k}) \quad (4)$$

Where $\gamma_{t-x}^{g,k}$ denotes a cohort effect.
$N(\mu, \tau)$: Denotes a normal distribution with a mean of μ and precision (inverse variance) τ.
The normal distribution assumed for rates is an extension of the Czado et al. (2005) model. It allows capturing the overdispersion that is not explained by the variability resulting from the Poisson sampling of count data.

General framework for forecasting mortality, fertility, emigration and immigration A. Wisniowski(2015)

Third. Time-specific effects $\kappa_t^{g,k}$. Cohort effect $\gamma_{t-k}^{g,k}$. They used time series model, which facilitates the forecasting

- To ensure identification of the parameters ($\alpha_x^{g,k}$, $\beta_x^{g,k}$, $\kappa_t^{g,k}$, $\gamma_{t-x}^{g,k}$. The following constraints are imposed.

$$\sum_{x=0}^{z} \beta_x^{g,k} = 1, \kappa_1^{g,k} = 0, \gamma_1^{g,k} = 0 \tag{5}$$

Where z denotes oldest age group.
These constraints suffice to identify the bilinear model in Eq. (4).

Sources

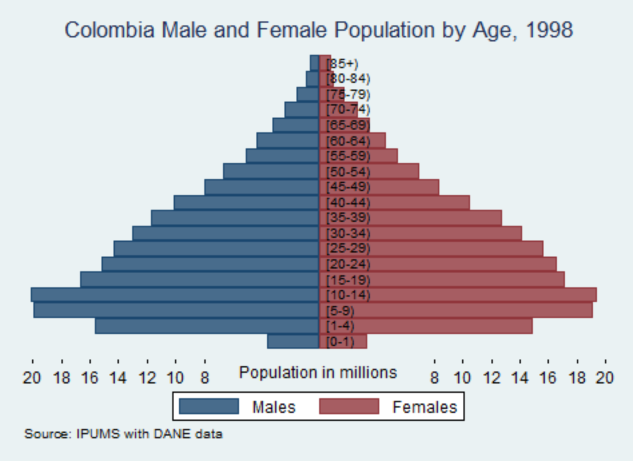
Demography

- Colpensiones
- AFPs
- Integrated Public Use Microdata Series (IPUMS): Census
- Dane: Vital Statistics

Economic Variables and Pension Schemes

- DNP
- Laws: Legis

Population



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Source: IPUMS with DANE data

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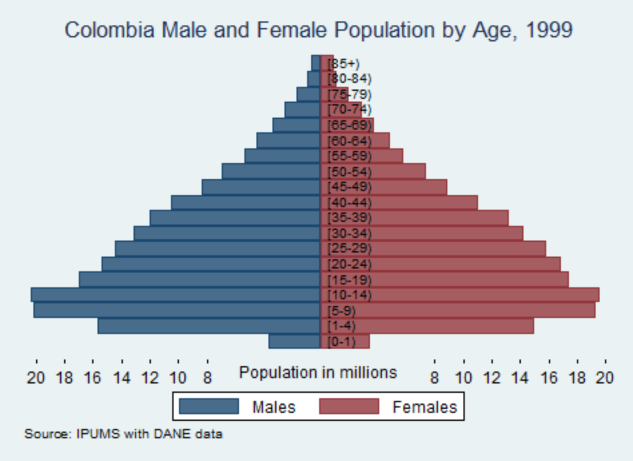

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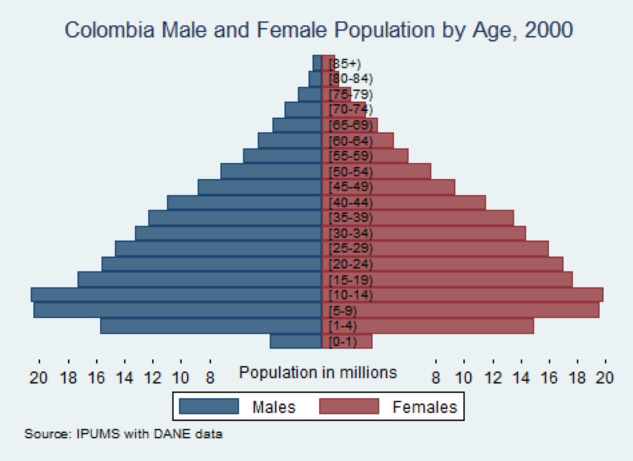
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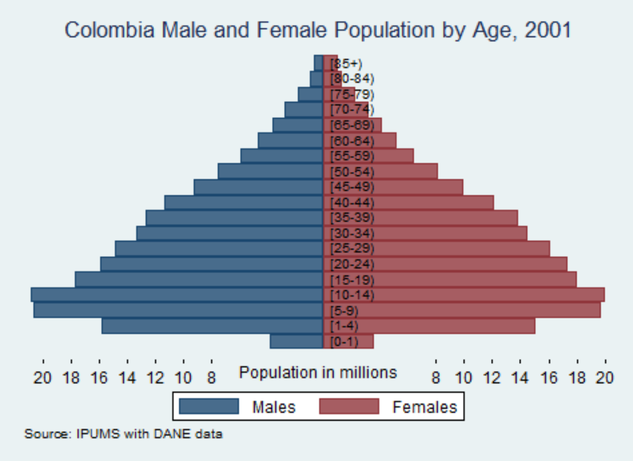

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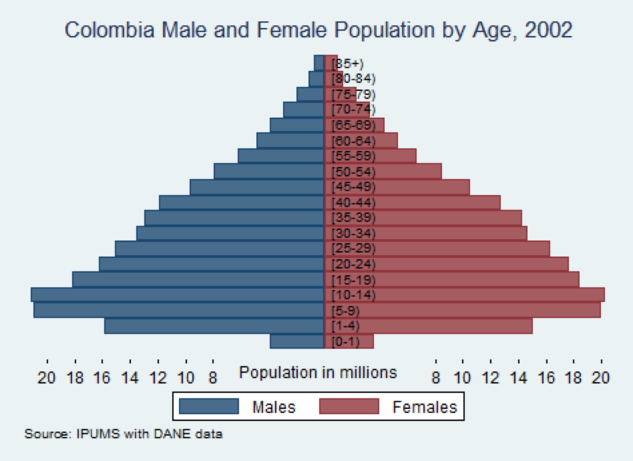

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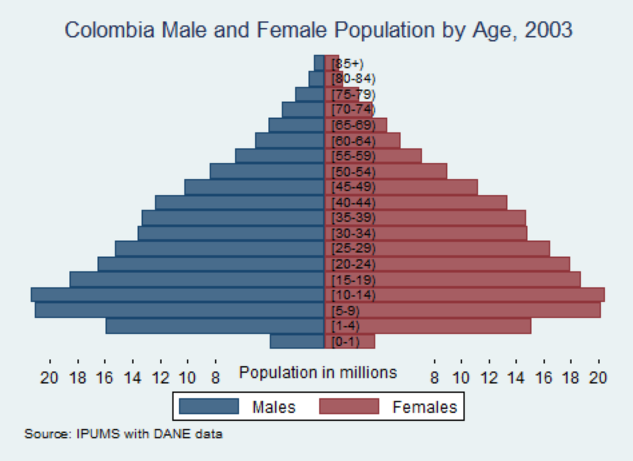

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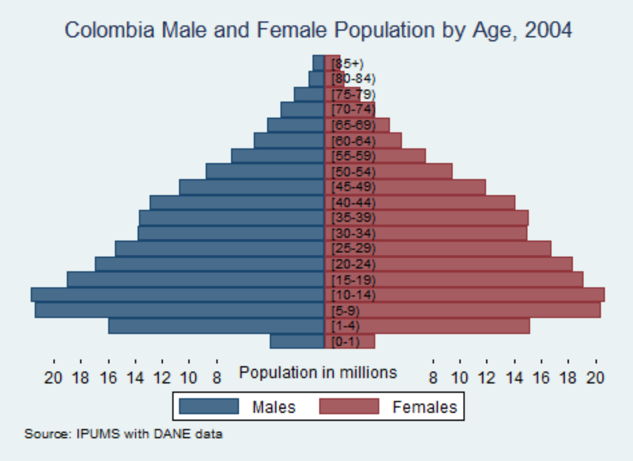

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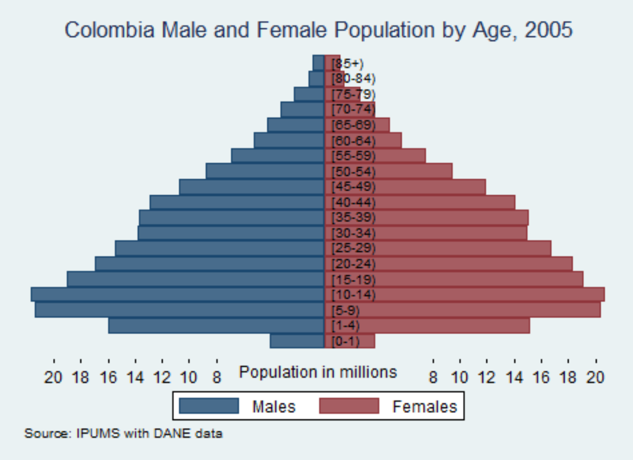

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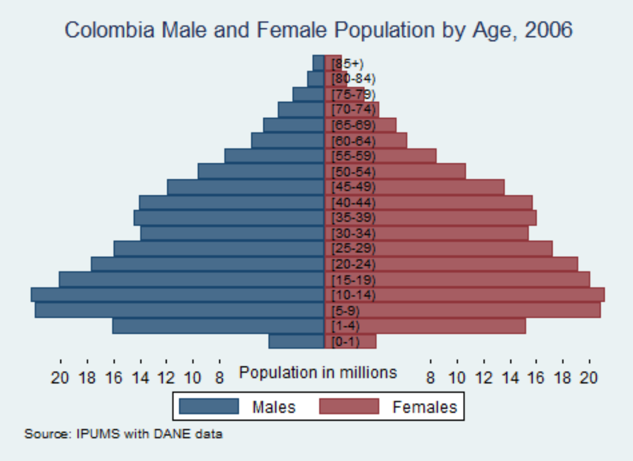

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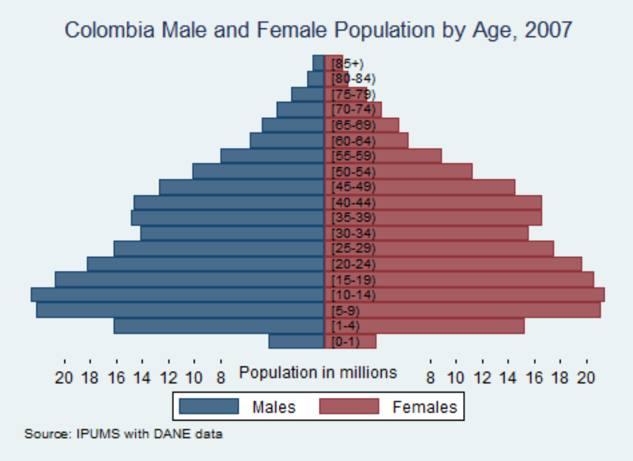

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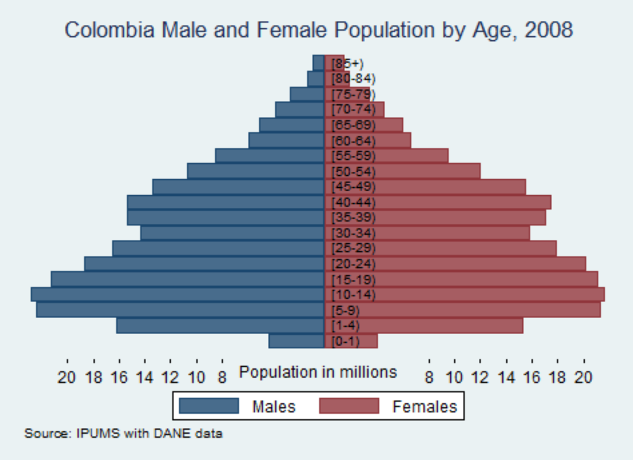
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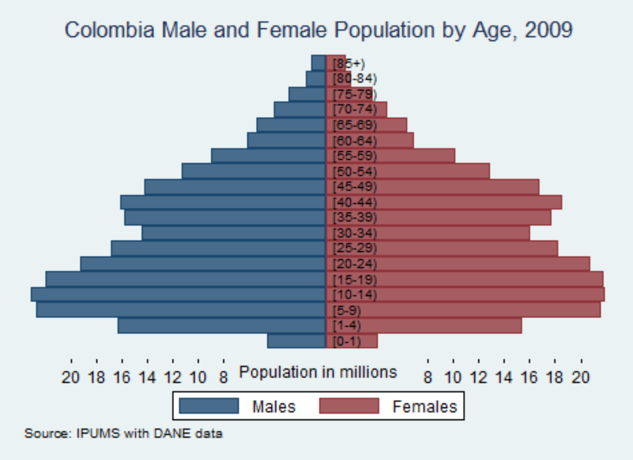

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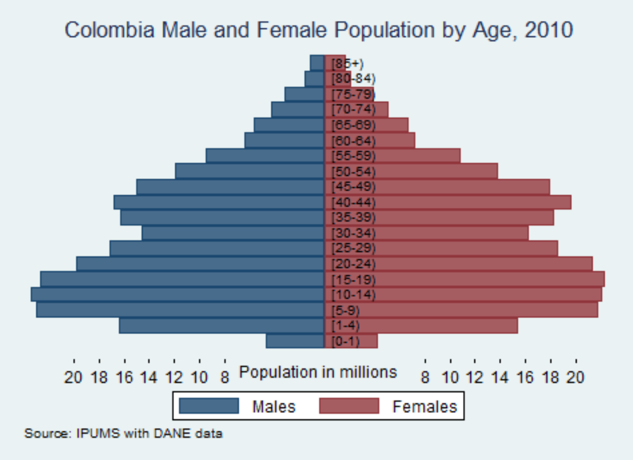

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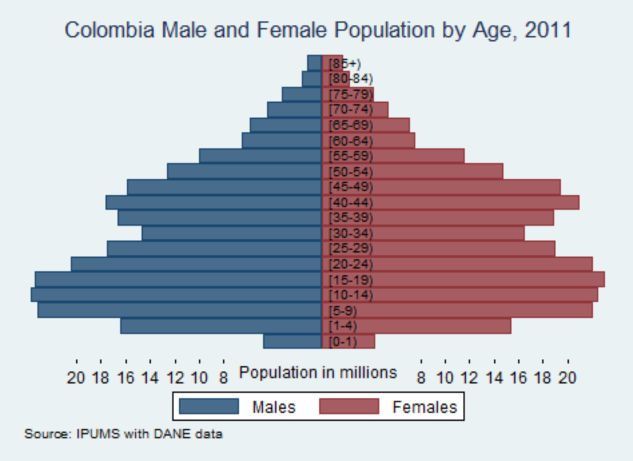

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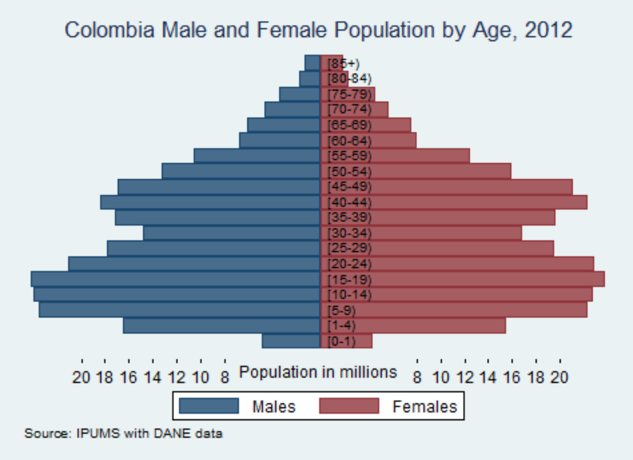

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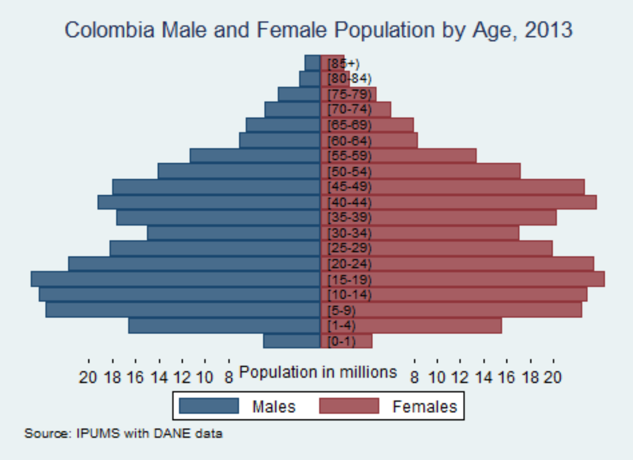
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The Lee-Carter model was originally designed to forecast age-specific mortality rates with the following specification

$$log\,\mu_{x,t} = \alpha_x + \beta_x \kappa_t + \xi_{x,t} \tag{6}$$

$$\kappa_t = \phi + \kappa_{t-1} + \varepsilon_t \tag{7}$$

Where x denotes age. t : time
$\mu_{x,t}$: Mortality rate. α_x : Age profile
β_x : How fast the rates decline over time in response of changes of κ_t
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$xi_{x,t}$: Error term. Assumed $\sim N(0, \sigma^2)$
To ensure identifiability of the model parameters $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$

Lee-Carter model M1

- The Lee-Carter model M1 is specified as in Eqs. (6) and (7),
- Age-specific mortality rates are calculated as:

$$\mu_{x,t}^{D,k} = Y_{x,t}^{D,k} / R_{x,t}^{D,k} \tag{8}$$

- Time-specific parameters κ_t^k follow univariate random walk with drift models

Lee (1993) F1

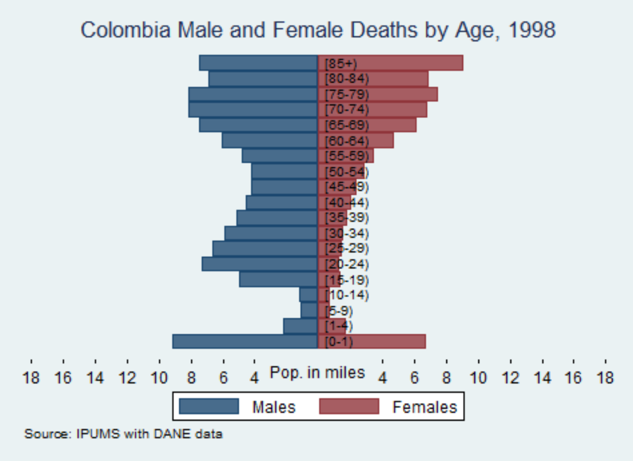
- The population at risk represents all women of reproductive age.
- The time component follows an ARMA (1,1) process.

$$\kappa_t \sim \mathbf{N}(\phi_0 + \phi_1(\kappa_{t-1} - \phi_0) + \phi_2 \xi_{t-1}, \tau_k) \tag{9}$$

Where

$$\xi_t = \kappa_t - \phi_0 - \phi_1(\kappa_{t-1}, \tau_\kappa) \tag{10}$$

Demography - Deaths



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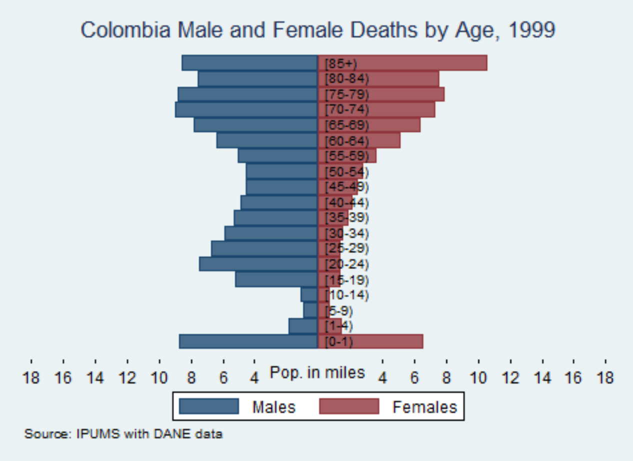

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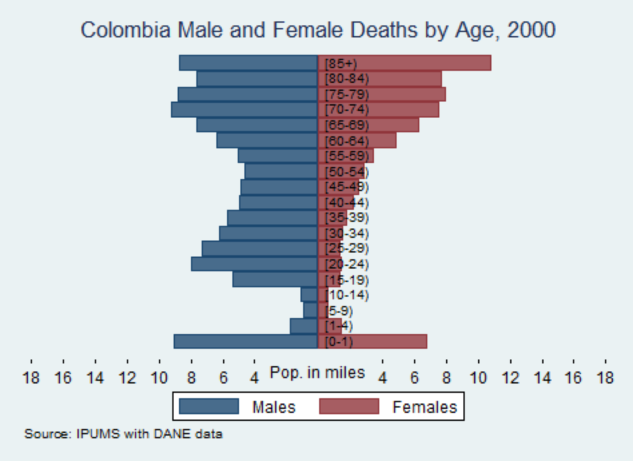

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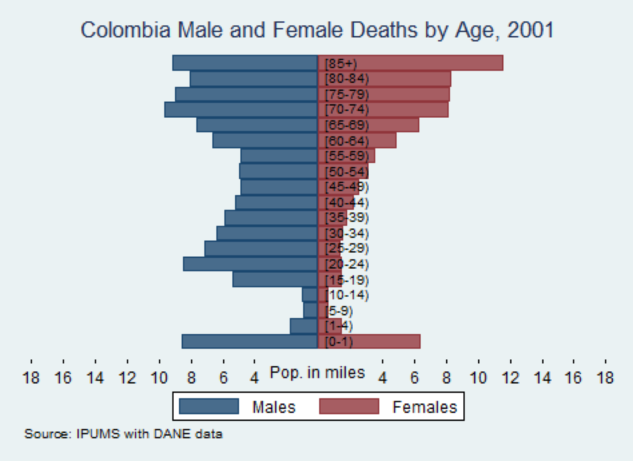

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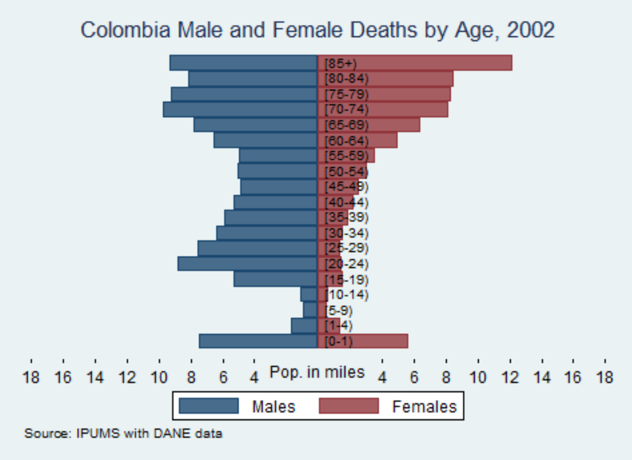

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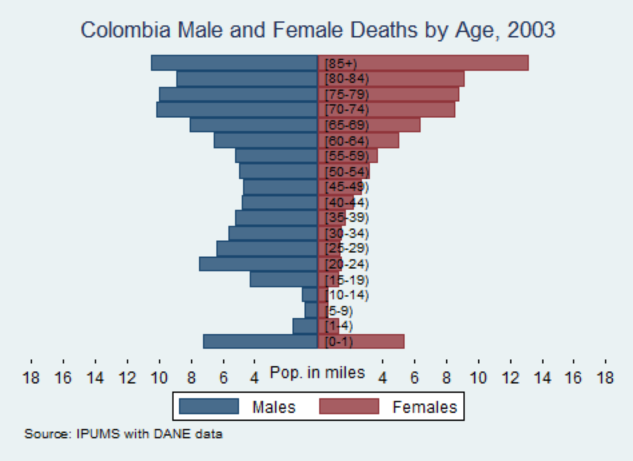

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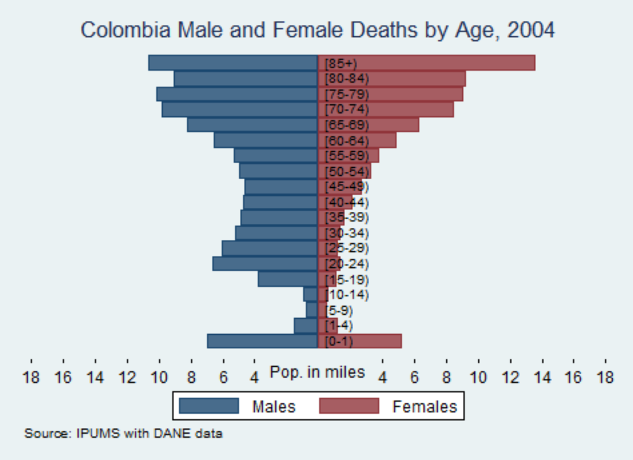

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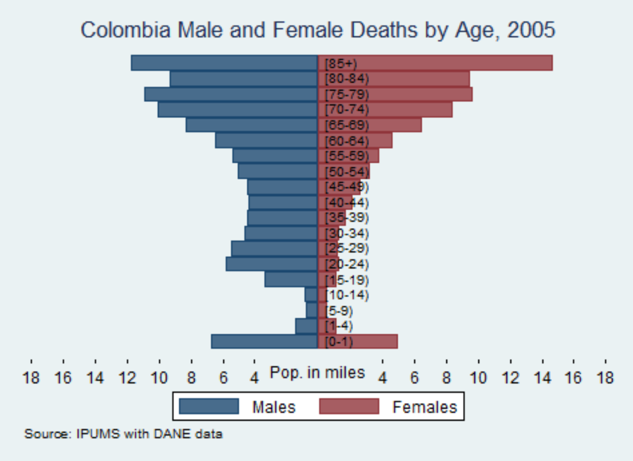

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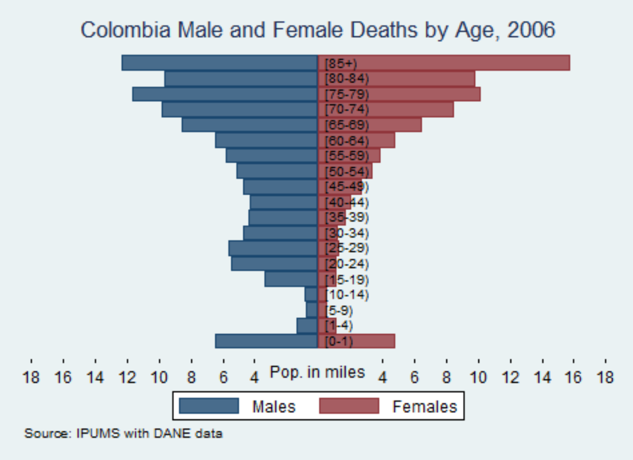

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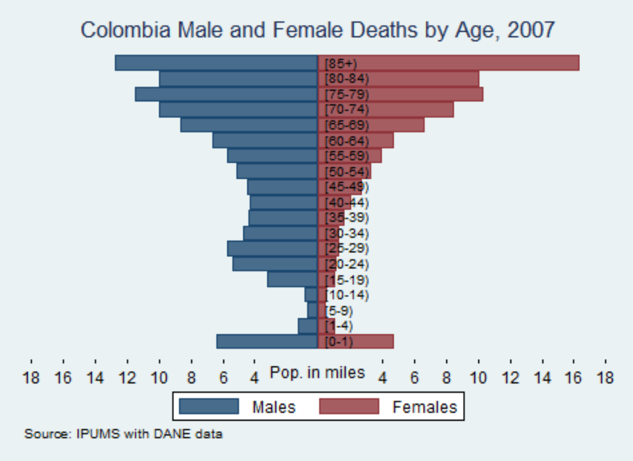

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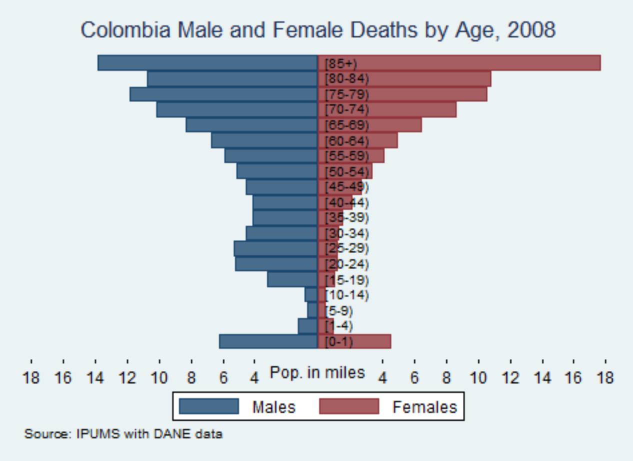
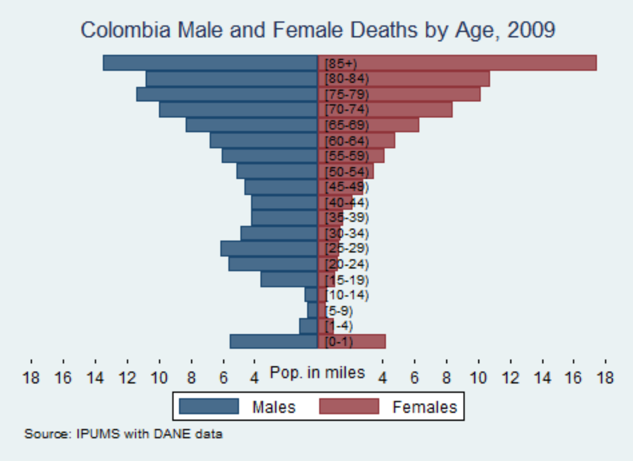
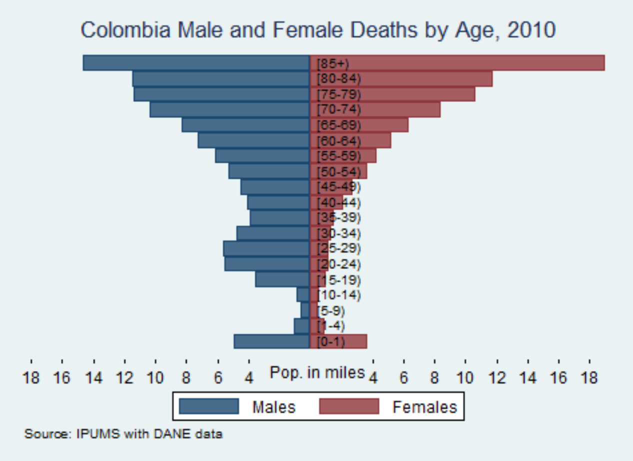


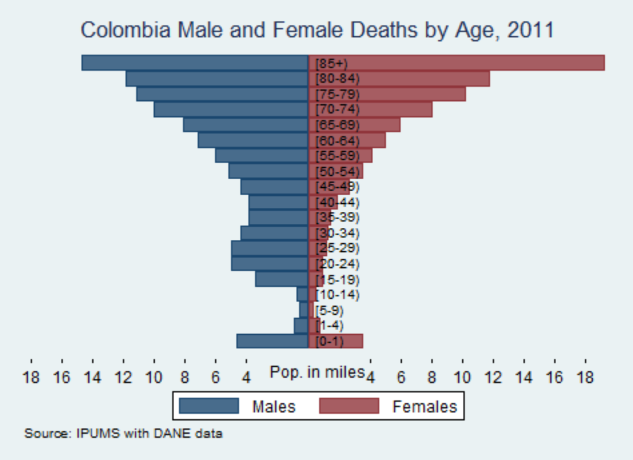
Demography - Deaths



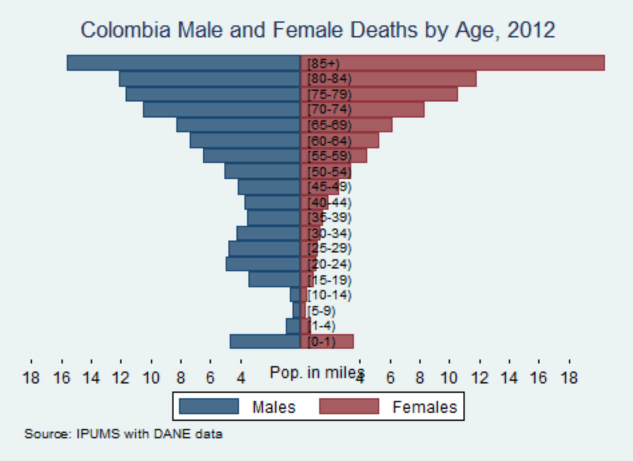
Demography - Deaths

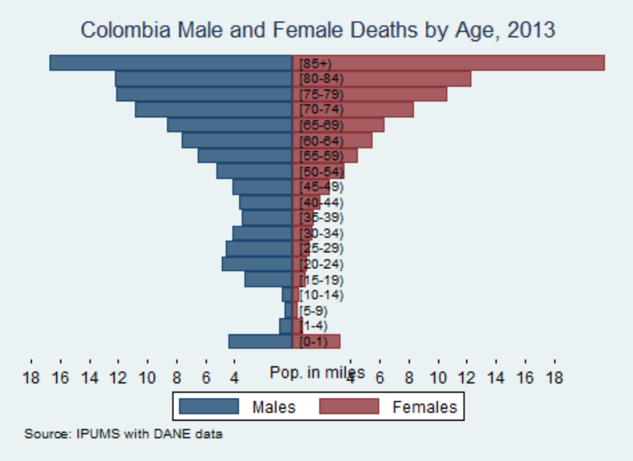


Demography - Deaths



Demography - Deaths




Colombia Male and Female Deaths by Age, 2013
[85+)
[80-84)
[75-79)
[70-74)
[65-69)
[60-64)
[55-59)
[50-54)
[45-49)
[40-44)
[35-39)
[30-34)
[25-29)
[20-24)
[15-19)
[10-14)
[5-9)
[1-4)
[0-1)
18 16 14 12 10 8 6 4 4 6 8 10 12 14 16 18
Pop. in miles
Males
Females
Source: IPUMS with DANE data

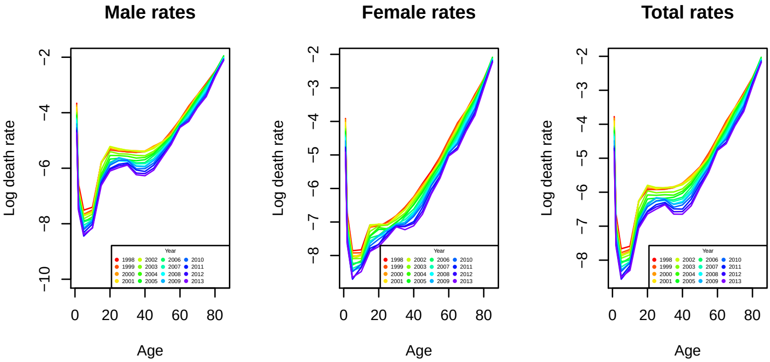


Figure: Mortality rate by age

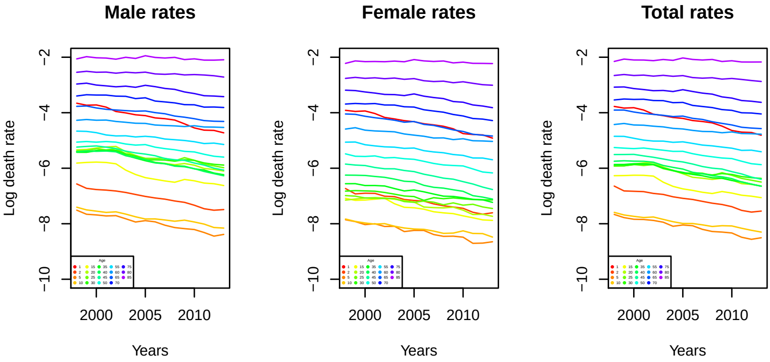


Figure: Mortality rate by years

Mortality Model - LC Parameters

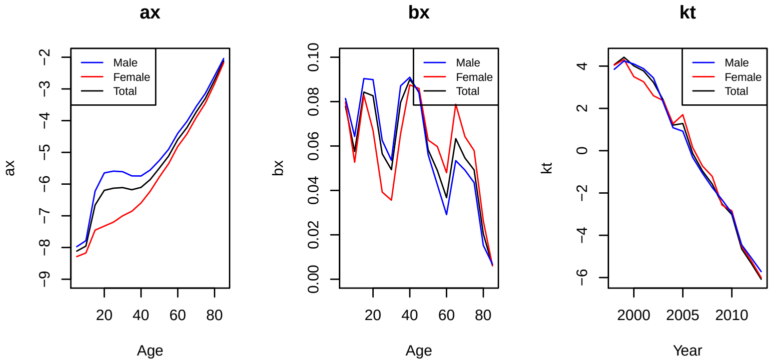


Figure: LC Parameters

Mortality Model - Forecast

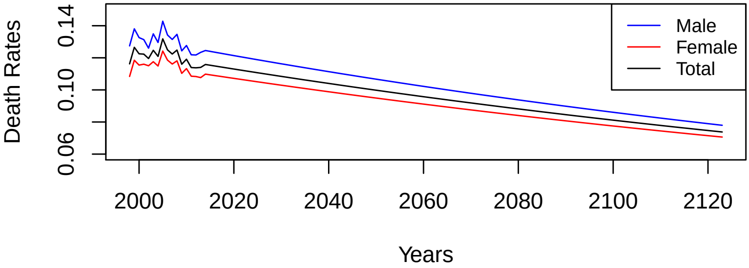


Figure: Death rates forecast

Fertility Model

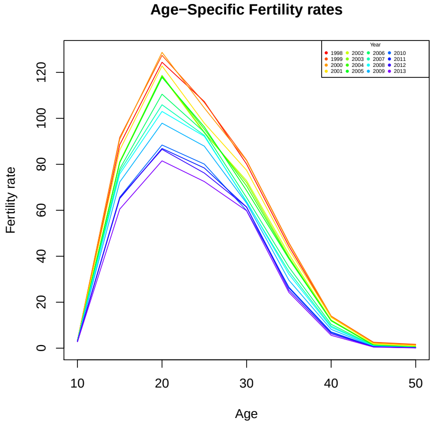


Figure: Fertility rate by age

Fertility Model

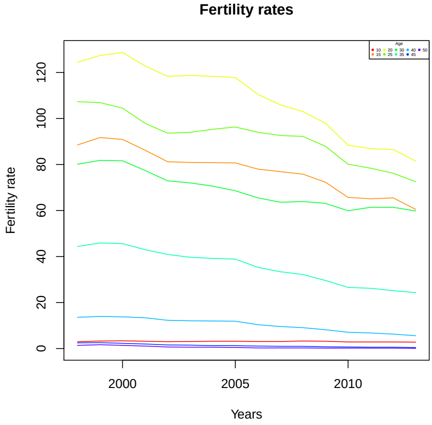


Figure: Fertility rate by year

Fertility Model - LC Parameters

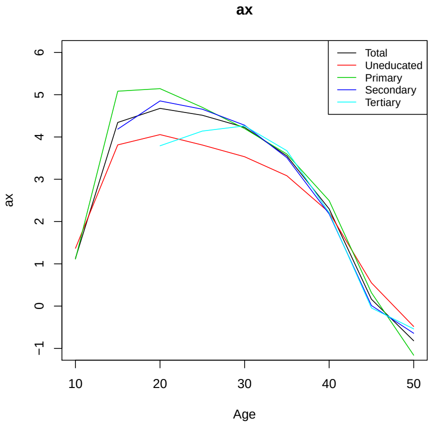


Figure: LC Parameters - ax

Fertility Model - LC Parameters

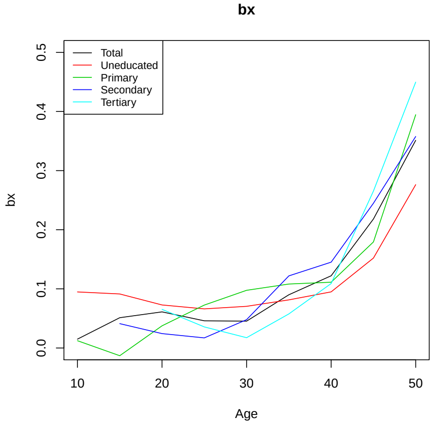


Figure: LC Parameters - bx

Fertility Model - LC Parameters

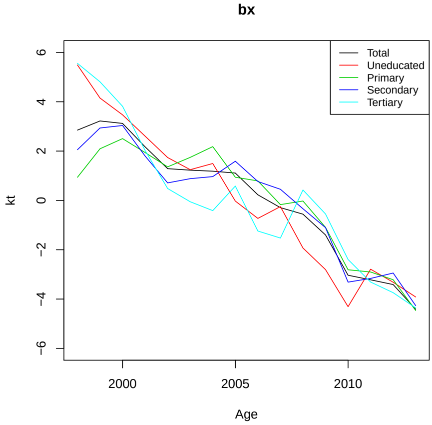


Figure: LC Parameters -kt

Fertility Model - Forecast

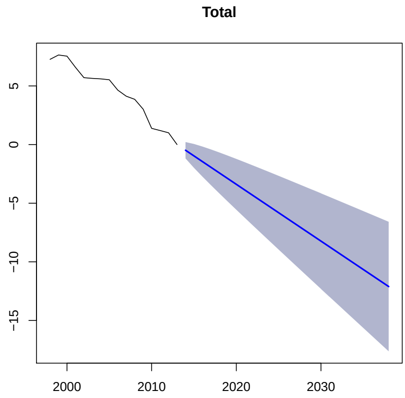


Figure: LC Forecast

Table: Males

Tabla de Mortalidad Colombia											
Resolución R1555 - 30 de Julio de 2010-Experiencia 2003-2008						DNP					
x	lx	dx	qx	px	e0	x	mx	qx	lx	dx	ex
0	1023050	16470	0,0161	0,9839	**78,1**	0	0,0141	0,0139	1,0000	0,0139	**75,3**
1	1006580	1290	0,0013	0,9987	**78,3**	1	0,0008	0,0031	0,9861	0,0031	**75,3**
5	1003720	390	0,0004	0,9996	**74,6**	5	0,0003	0,0015	0,9830	0,0015	**71,5**
10	1001910	330	0,0003	0,9997	**69,7**	10	0,0004	0,0018	0,9816	0,0018	**66,6**
15	1000000	485	0,0005	0,9995	**64,8**	15	0,0015	0,0075	0,9798	0,0073	**61,8**
20	997451	553	0,0006	0,9994	**60,0**	20	0,0028	0,0139	0,9725	0,0136	**57,2**
25	994488	662	0,0007	0,9993	**55,1**	25	0,0032	0,0160	0,9589	0,0154	**53,0**
30	990868	832	0,0008	0,9992	**50,3**	30	0,0032	0,0158	0,9435	0,0149	**48,8**
35	986216	1102	0,0011	0,9989	**45,6**	35	0,0027	0,0135	0,9286	0,0125	**44,5**
40	979936	1525	0,0016	0,9984	**40,8**	40	0,0027	0,0132	0,9161	0,0121	**40,1**
45	971105	2186	0,0023	0,9977	**36,2**	45	0,0034	0,0167	0,9040	0,0151	**35,6**
50	958298	3213	0,0034	0,9966	**31,6**	50	0,0048	0,0238	0,8889	0,0211	**31,2**
55	939348	4744	0,0051	0,9949	**27,2**	55	0,0069	0,0341	0,8677	0,0296	**26,9**
60	911595	6988	0,0077	0,9923	**23,0**	60	0,0114	0,0554	0,8382	0,0465	**22,7**
65	869557	11080	0,0127	0,9873	**19,0**	65	0,0162	0,0780	0,7917	0,0618	**18,9**
70	802940	16972	0,0211	0,9789	**15,3**	70	0,0261	0,1228	0,7299	0,0896	**15,3**
75	704342	23970	0,0340	0,9660	**12,1**	75	0,0425	0,1928	0,6403	0,1234	**12,1**
80	570538	30646	0,0537	0,9463	**9,3**	80	0,0746	0,3164	0,5169	0,1635	**9,3**
85	408381	34093	0,0835	0,9165	**7,0**	85+	0,1344	1,0000	0,3533	0,3533	**7,4**

Table: Females

Tabla de Mortalidad Colombia											
Resolución R1555 - 30 de Julio de 2010-Experiencia 2003-2008						DNP					
x	lx	dx	qx	px	e0	x	mx	qx	lx	dx	ex
0	1016720	12200	0,0120	0,9880	83,7	0	0,0112	0,0111	1,0000	0,0111	81,7
1	1004520	1000	0,0010	0,9990	83,7	1	0,0006	0,0025	0,9889	0,0025	81,6
5	1002280	260	0,0003	0,9997	79,9	5	0,0002	0,0011	0,9864	0,0011	77,8
10	1001090	200	0,0002	0,9998	75,0	10	0,0002	0,0012	0,9853	0,0012	72,9
15	1000000	272	0,0003	0,9997	70,0	15	0,0005	0,0025	0,9841	0,0024	68,0
20	998570	311	0,0003	0,9997	65,1	20	0,0006	0,0030	0,9817	0,0029	63,1
25	996905	372	0,0004	0,9996	60,2	25	0,0007	0,0033	0,9788	0,0033	58,3
30	994869	469	0,0005	0,9995	55,4	30	0,0008	0,0041	0,9755	0,0040	53,5
35	992247	622	0,0006	0,9994	50,5	35	0,0009	0,0045	0,9715	0,0044	48,7
40	988699	863	0,0009	0,9991	45,7	40	0,0012	0,0060	0,9671	0,0058	43,9
45	983694	1242	0,0013	0,9987	40,9	45	0,0017	0,0086	0,9613	0,0083	39,2
50	976402	1836	0,0019	0,9981	36,2	50	0,0028	0,0138	0,9530	0,0132	34,5
55	965536	2735	0,0028	0,9972	31,6	55	0,0043	0,0214	0,9398	0,0202	29,9
60	949454	4082	0,0043	0,9957	27,0	60	0,0074	0,0363	0,9197	0,0334	25,5
65	924968	6351	0,0069	0,9931	22,7	65	0,0107	0,0522	0,8863	0,0463	21,4
70	886485	10065	0,0114	0,9886	18,6	70	0,0187	0,0893	0,8400	0,0750	17,4
75	825554	15832	0,0192	0,9808	14,7	75	0,0305	0,1420	0,7649	0,1086	13,9
80	731048	23943	0,0328	0,9672	11,3	80	0,0571	0,2512	0,6564	0,1648	10,7
85	592620	33252	0,0561	0,9439	8,3	85+	0,1181	1,0000	0,4915	0,4915	8,5

A Demographic Model. DNpension

Felipe Sánchez

Dirección de Estudios Económicos

March, 2017

Demography

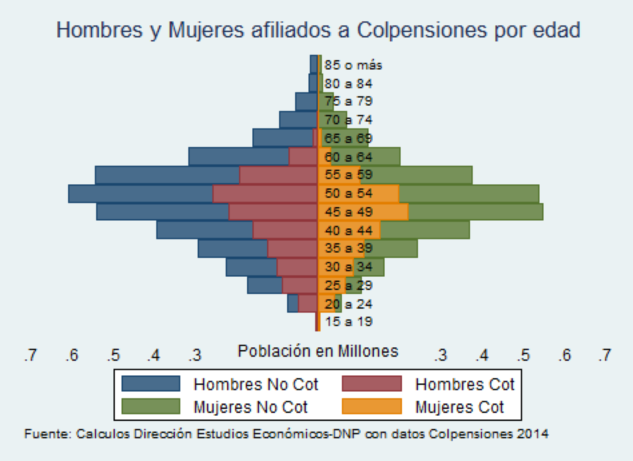


Figure: Affiliated Colpensiones -Actives, Inactives

Demography

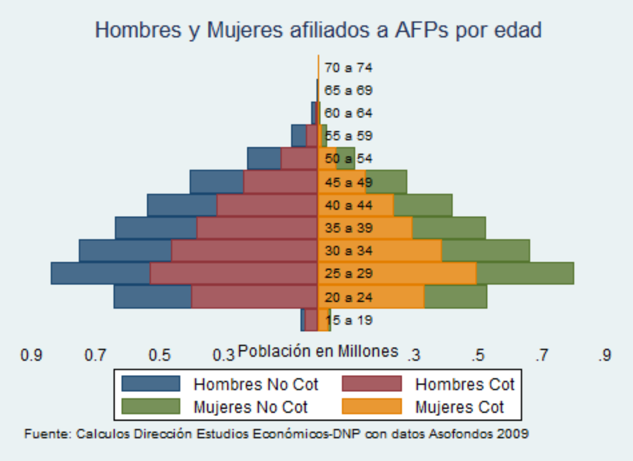


Figure: Affiliated AFPs

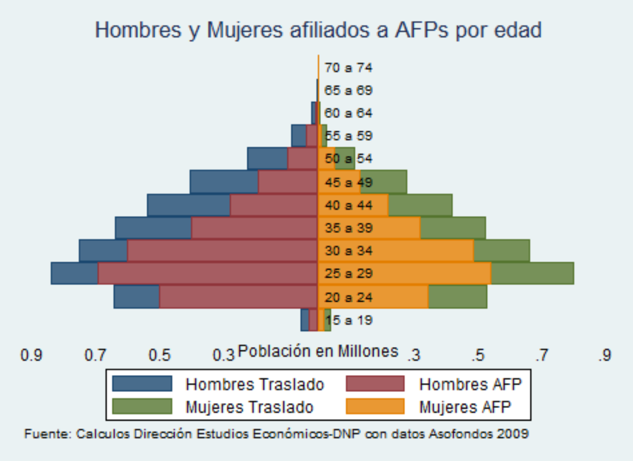


Figure: Affiliated AFPs -Transfers

Males		Females	
Affiliated	Active	Affiliated	Active
7.214.699	4.044.452	5.318.653	2.878.420

Table: Affiliated and Active AFPs. 2014

Males		Females	
Affiliated	Active	Affiliated	Active
3.541.373	1.298.045	2.864.105	1.029.293

Table: Affiliated and Active Colpensiones. 2014